Math 53, Discussion 107

Friday, April 3, 2020 9:46 AM

1. Describe what the surface $\phi = \frac{\pi}{3}$ looks like
2. Evaluate $\iiint_E e^{\sqrt{x^2+y^2+z^2}}\,dV$ where E is enclosed by the sphere $x^2+y^2+z^2=9$ and the first octant
3. Set up the determinant to determine the integration constant for integrating in spherical coordinates
4. Find the Jacobian of the transformation $x = uv, y = \frac{u}{v}$
5. Compute $\iint_R x^2\,dA$ where R is the region bounded by the ellipse $9x^2+4y^2=36$, (use the sub $x = 2u, y = 3v$)

1) Evaluate $\int_0^1\int_0^1 e^{\max(x^2,y^2)}\,dy\,dx$

2) Find the average value of $f(x) = \int_x^1 \cos(t^2)\,dt$ on the interval [0,1]

3) Prove $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$ by the following:

A) using $\sum x^n = \frac{1}{1-x}$ show that $\int_0^1\int_0^1 \frac{1}{1-xy}\,dx\,dy = \sum_1^\infty \frac{1}{n^2}$

B) evaluate the integral with sub $x = \frac{u-v}{\sqrt{2}}, y = \frac{u+v}{\sqrt{2}}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad 0 \le x < 1$

$\int_0^1\int_0^1 \frac{1}{1-xy}\,dx\,dy = \int_0^1\int_0^1 \sum x^n y^n\,dx\,y = \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^3} = ???$

The joint density function for a pair of random variables X and Y is $f(x,y) = Cx(1+y)$ if $0 \le x \le 1, 0 \le y \le 2$ and zero otherwise.

A) find the value of the constant C

B) find $P(X \le 1, Y \le 1)$

C) find $P(X+Y \le 1)$

Def Probability X & Y take values in a region R

$= \iint_R f(x,y)\,dx\,dy$

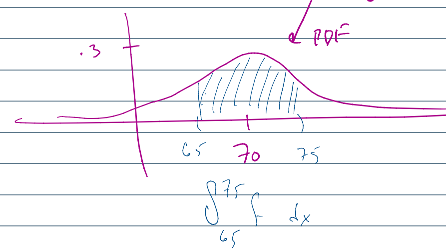
Require $\iint_{\mathbb{R}^2} f\,dA = 1$

in 1 variable

$P(a \le X \le b) = \int_a^b f(x)\,dx$

Probability density

PDF



$\int_{65}^{75} f\,dx$

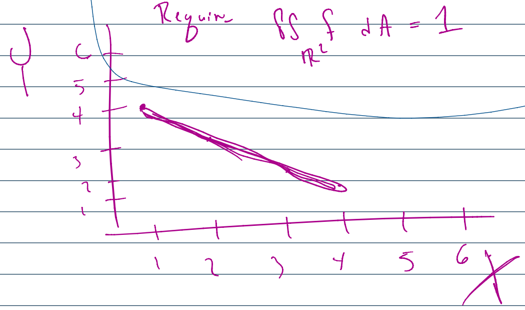
a) need $\iint f = 1$

$1 = \int_0^2\int_0^1 c\,x(1+y)\,dx\,dy$

$= c\int_0^2 (1+y)\,\frac{x^2}{2}\Big|_{x=0}^{x=1}$

$= \frac{c}{2}\left(y + \frac{y^2}{2}\Big|_0^2\right) = \frac{4c}{2}$

$\Rightarrow c = \frac{1}{2}$

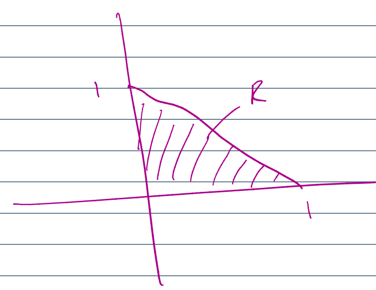


$P(X+Y \le 1) = \iint_R f(x,y)\,dx\,dy$

R is the region where $X+Y \le 1$

$Y \le 1-x$

$= \int_0^1\int_0^{1-x} \frac{1}{2}x(1+y)\,dy\,dx$



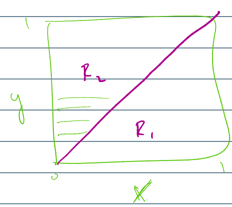
$\int_0^1\int_0^1 e^{\max(x^2,y^2)}\,dx\,dy$

$\iint_{R_1} e^{x^2}\,dy\,dx + \iint_{R_2} e^{y^2}\,dx\,dy$

$\int_0^1\int_0^x e^{x^2}\,dy\,dx + \int_0^1\int_0^y e^{y^2}\,dx\,dy$

$= 2\int_0^1\int_0^x e^{x^2}\,dy\,dx$

$= 2\int_0^1 x e^{x^2}\,dx = e^{x^2}\Big|_0^1 = e-1$



function